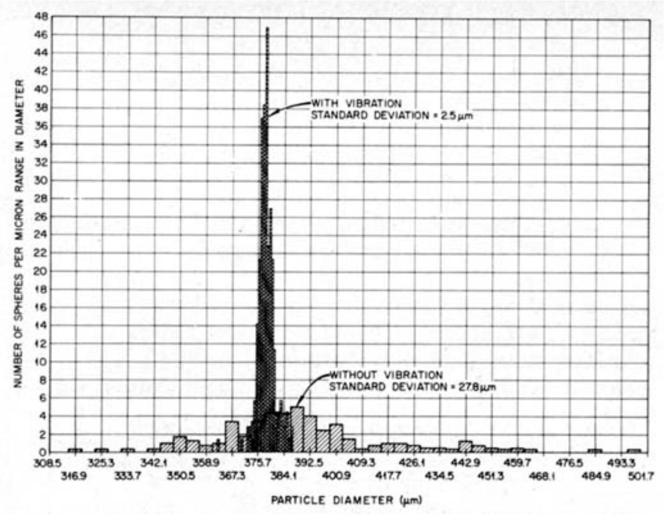

Fig. 2. Comparison of size distribution of ThO_2 microspheres made with and without vibration.

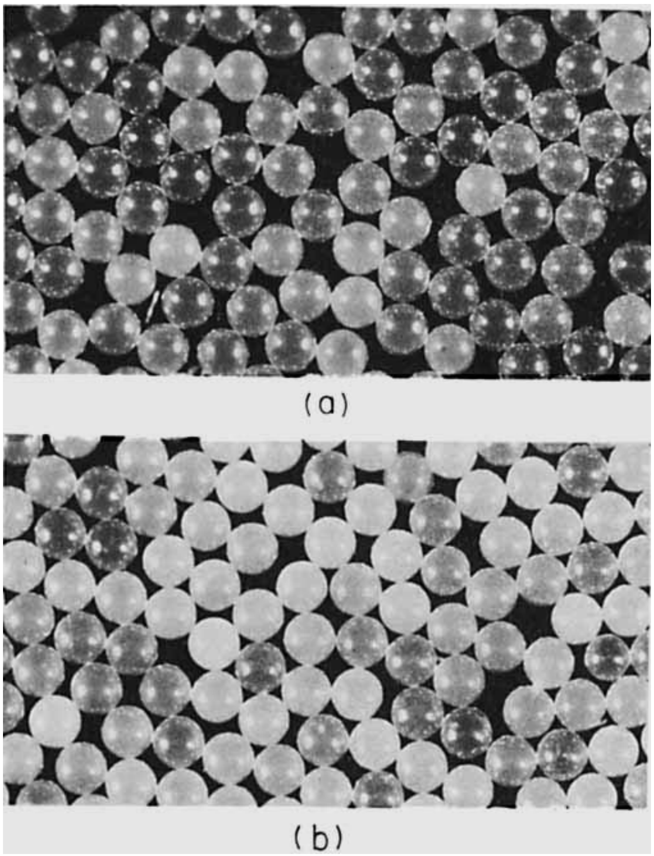

Fig. 3. Photomicrographs of samples riffled from (a) Batch J-146 and (b) Batch J-147.

employing vibration were extremely uniform in size. The standard deviation of the particle diameter was 2.5 μm, compared with 27.8 μm for spheres made without vibration. Typical test conditions and results for other batches are tabulated in Table 2.

The densities of the spheres comprising four of the batches (J-146, -147, -148, and -149) were determined by mercury pycnometry. In each of these batches, the spheres were found to be theoretically dense to within the accuracy of the technique (0.1 g/cm^3). Photographs of riffled samples from two batches are shown in Figure 3.

ACKNOWLEDGMENT

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Laminar Two-Dimensional Non-Newtonian Jets

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Laminar Newtonian jets have been studied extensively and a full account of all significant theoretical and experimental work is given by Schlichting (1968). The boundary layer theory and the similarity solutions given by Schlichting have also been extended to laminar non-Newtonian jets. Lemieux and Unny (1968) and Atkinson (1972) developed similarity solutions for the two-dimensional jet and Rotem (1964) for the axisymmetric jet of a power-law fluid issuing into a mass of the same fluid. Laminar non-Newtonian jets are perhaps more realistic than Newtonian. Many non-Newtonian fluids, such as polymer solutions and melts, have large apparent viscosities and laminar flow persists under usual processing conditions. A study of jets of such fluids is of basic interest in polymer mixing, extrusion, injection molding, and drag reduction. Actually polymer solutions and melts exhibit elastic effects in addition to non-Newtonian behavior. Accordingly a complete study should include normal stress differences. In the present paper an attempt in that direction is made by examining first the simpler inelastic problem. The present numerical method was developed because the existing similarity solutions are valid only in large distances from the jet orifice although in the chemical engineering applications mentioned above the early part of the flow is more important.

MATHEMATICAL PROCEDURE

For the purpose of mathematical analysis the following assumptions are made:

1. The fluid is incompressible and obeys the Ostwald-De Waele power law with a constant consistency index m and power law index n ($n < 1$ for pseudoplastics).

$$\tau = m \left| \frac{\partial u}{\partial y} \right|^{n-1} \frac{\partial u}{\partial y} \tag{1}$$

2. The flow is steady.
3. There are no end effects or gravitational forces.
4. The jet is issuing from a slit orifice with a large aspect ratio into a mass of the same otherwise undisturbed fluid.

The boundary layer equations for this laminar, two-dimensional jet are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = m \frac{\partial}{\partial y} \left(\left| \frac{\partial u}{\partial y} \right|^{n-1} \frac{\partial u}{\partial y} \right) \tag{3}$$

In terms of dimensionless variables, the equations are

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{4}$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \frac{\partial}{\partial Y} \left\{ \left| \frac{\partial U}{\partial Y} \right|^{n-1} \frac{\partial U}{\partial Y} \right\} \tag{5}$$

The boundary conditions are

$$X = 0 \qquad U = U_i(0, Y) \qquad V = 0 \tag{6}$$

$$Y = 0 \qquad \frac{\partial U}{\partial Y} = 0 \qquad V = 0 \tag{7}$$

$$Y \to \infty \qquad U \to 0 \tag{8}$$

Solutions will be obtained for the following two initial profiles ($X = 0$).

(1) Rectangular

$$U_i = 1 \quad Y \leq 1/2; \qquad U_i = 0 \quad Y > 1/2 \tag{9}$$

(2) Nearly parabolic (corresponds to fully developed flow between two parallel plates)

$$U_i = 1 - \left(\frac{2Y}{D} \right)^{\frac{n+1}{n}} \quad Y \leq 1/2; \qquad U_i = 0 \quad Y > 1/2 \tag{10}$$

A finite-difference method developed by Tomich and Weger (1967) for axisymmetric, compressible, turbulent-free jets was modified and adapted to the present two-dimensional, non-Newtonian jet. A network was constructed (x-direction of flow, y-perpendicular) and the following finite difference approximations at ($X + \Delta X$, Y) were introduced to the dimensionless conservation Equations (4) and (5):

$$\frac{\partial U}{\partial X} = \frac{U(X + \Delta X, Y) - U(X, Y)}{\Delta X} \tag{11}$$

$$\frac{\partial U}{\partial Y} = \frac{U(X + \Delta X, Y + \Delta Y) - U(X + \Delta X, Y - \Delta Y)}{2\Delta Y} \tag{12}$$

$$\frac{\partial^2 U}{\partial Y^2} = \frac{U(X + \Delta X, Y + \Delta Y) - 2U(X + \Delta X, Y) + U(X + \Delta X, Y - \Delta Y)}{(\Delta Y)^2} \tag{13}$$

$$\frac{\partial V}{\partial Y} = \frac{V(X + \Delta X, Y) - V(X + \Delta X, Y - \Delta Y)}{\Delta Y} \tag{14}$$

Upon substitution into the momentum equation a system of nonlinear algebraic equations was obtained which was linearized and solved in an iterative scheme until sufficient accuracy was obtained. The solution was then carried downstream to line $X + 2\Delta X$ and so forth (marching procedure). The V velocities were calculated explicitly from the finite-difference form of the equation of continuity.

The boundary condition at $Y \to \infty$ was realized by setting U equal to zero at some large distance from the jet centerline. Normally about 25 nozzle diameters from the centerline were sufficient. To ensure convergence of the numerical scheme to the true solution, various step sizes, step-size ratios, and locations of the outer jet boundary ($Y \to \infty$) were tried. The results reported here are independent of these variables within a small tolerance.

An additional indication of the accuracy of the present numerical technique was obtained by comparing the results to Pai's (1972) numerical calculations for Newtonian jets. The velocity profiles U, the transverse velocity V, and the half-jet width $b_{1/2}$ were virtually indistinguishable for both rectangular and parabolic profiles.

There were no specific problems in extending Tomich and Weger's method to the non-Newtonian case except those caused by the fact that the power-law expression gives infinite apparent viscosity when $\partial u/\partial y \to 0$ and $n < 1$. To circumvent this difficulty it was assumed that the apparent viscosity $[\tau/(\partial u/\partial y)]$ is constant (maximum) when the dimensionless velocity gradient $\partial U/\partial Y < 10^{-5}$. Constancy of the apparent viscosity at low shear rates (velocity gradients) is in agreement with experimental data for polymer solutions and melts. [Here, it must be noted that virtually all numerical calculations that involve the equation of momentum for a power-law fluid with $n < 1$ require the specification of a maximum apparent viscosity. An extensive literature search revealed only one paper, by Ozoe and Churchill (1972), in which such a maximum value is specified.]

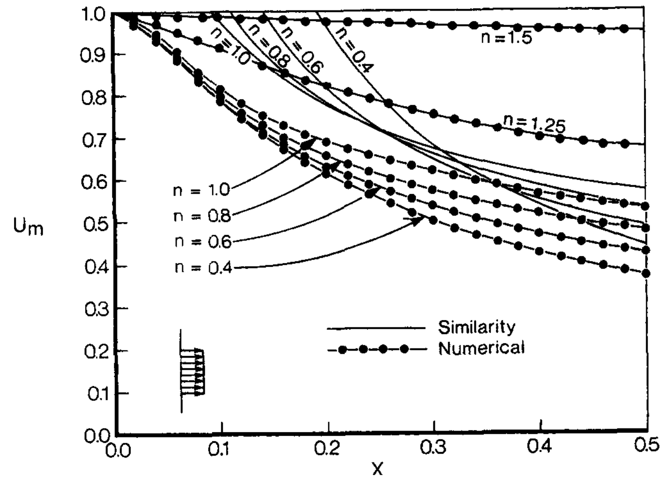

Fig. 1. Maximum velocity decay along the jet midplane for a rectangular velocity prcfile at the orifice.

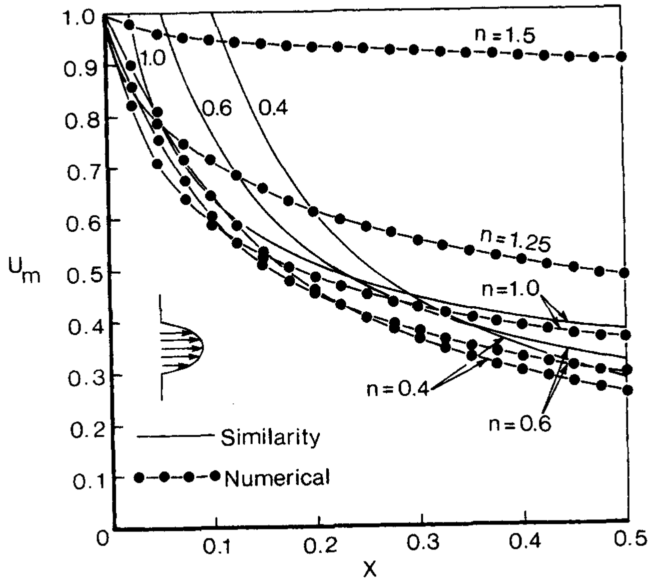

Fig. 2. Maximum velocity decay along the jet midplane for nearly parabolic velocity profiles at the orifice.

RESULTS AND DISCUSSION

The velocity drop along the midplane of the jet (maximum) is shown in Figure 1 for the rectangular profile at the orifice, and in Figure 2 for the parabolic profile. From Figure 1 we can see that the maximum velocity at any given location decreases as the power-law index n decreases. This is physically explained by the increase in apparent viscosity of the fluid as n decreases. In Figure 2 the maximum velocity decay curves cross each other at the early part of the flow. This is due to the fact that the velocity profile at the orifice, as given by Equation (10) produces a jet of larger momentum as the power-law index decreases (that is, same maximum velocity u_0 but flatter velocity profiles at $X = 0$).

The numerical results are compared to Lemieux and Unny's (1968) analysis for pseudoplastic fluids, which is a generalization of Schlichting's (1968) similarity solution. This analysis stipulates a jet of infinitesimally small orifice diameter but possessing finite momentum (and thus infinitely large velocity) at the origin. As the distance from the orifice increases the similarity solution approaches asymptotically the numerical results. It is interesting to note that parabolic initial profiles give a closer agreement to the similarity solution than rectangular initial profiles.

The spreading of a submerged jet is best characterized by its half-jet width $b_{1/2}$ (that is, the locus of points for which $U = \frac{1}{2}U_m$). The half-jet width as a function of distance from the origin is shown in Figures 3 and 4. As the power-law index decreases, the jet spreading increases because of the increase in apparent viscosity. The crossing of the curves for the parabolic initial profiles is due to the initial momentum difference for different n values. No comparison is given to the similarity solution because of extremely large deviations. However, the similarity analysis does predict the correct shape of the jet spreading. Lemieux and Unny (1968) predict a jet expansion boundary to be concave to the jet axis for $n > 2/3$ and convex for $n < 2/3$, fully in agreement with the present results.

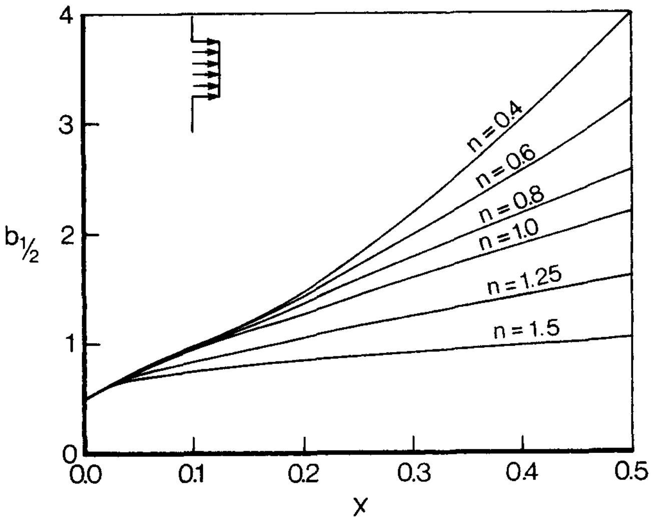

Fig. 3. Half-jet width for a rectangular velocity profile at the orifice.

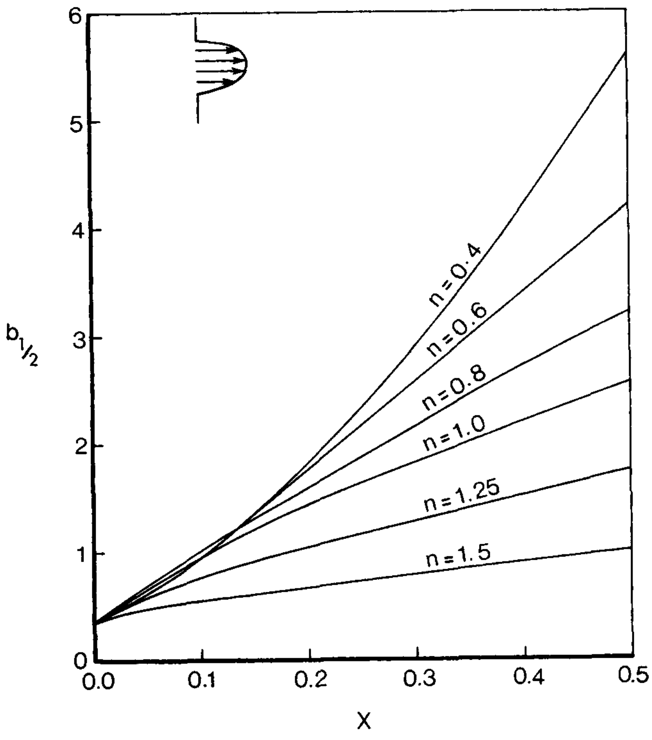

Fig. 4. Half-jet width for nearly parabolic velocity profiles at the orifice.

The numerical solution presented here is capable of describing non-Newtonian jet flow under realistic conditions (finite width orifice, any type of initial profiles) and has certain advantages over the similarity solution. However, in many instances it might be easier to use the similarity solution. It is possible to improve the results of the similarity analysis by defining a virtual origin at the jet, that is, shifting the origin of jet to fit experimental data or the present numerical results. Such an improvement has been suggested by Pai (1972), for Newtonian jets.

ACKNOWLEDGMENT

Financial assistance from the National Research Council of Canada is gratefully appreciated.

NOTATION

$b_{1/2}$ = half-jet width, that is, locus of points for which $U = \frac{1}{2}U_m$
D = orifice width
m = consistency index of the power law model
n = power law index
Re = Reynolds number, $D^n u_0^{2-n}\rho/m$
u = velocity in main flow direction x
u_0 = maximum velocity at the orifice
U = dimensionless velocity, $= u/u_0$
U_i = velocity profile at the orifice (initial)
U_m = velocity at the jet midplane
v = velocity in the y-direction
V = dimensionless velocity, $= vRe/u_0$
x = coordinate in the main flow direction
X = dimensionless coordinate, x/DRe
y = coordinate perpendicular to main flow direction
Y = dimensionless coordinate, y/D
ΔX = X-direction stepsize
ΔY = Y-direction stepsize
ρ = density
τ = shear stress

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Some Notes on the Temperature Response of Packed Beds

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Design and control problems in connection with various kinds of plants in which a bed of solid material is transported by a grid or grate while being processed by a cross flow of fluid have revived the interest in the responses of packed or porous beds (Voskamp et al., 1972). Some of these problems have already been discussed as early as in 1926 in connection with the heat recuperator (Anzelius, 1926) see also the survey by Jakob (1957), but changed conditions make it worthwhile to reconsider the problems. Kohlmayr (1968) did this a few years ago, presenting a new pair of analytical solutions claiming that they lend themselves to more efficient computation than Schumann's analytical solution, and that it is no longer necessary to resort to finite-difference methods when computing theoretical response functions.

The present paper submits another analytical solution based on a finite-difference approach of which the solution presented by Kohlmayr is a special case. Ironically, our solution was developed in 1968, but only recently did we discover that in digital computations it is more efficient than any other solution we have tried, except when an unusually high accuracy is desired.

BASIC EQUATIONS

The following set of equations describe the bed of spherical particles, heated or cooled by a fluid:

$$\frac{1}{\kappa}\frac{\partial T_s}{\partial t} = \frac{\partial^2 T_s}{\partial r^2} + \frac{2}{r}\frac{\partial T_s}{\partial r}, \quad \kappa = \lambda_s/(\rho_s\gamma_s) \tag{1}$$

$$T_s\{r, 0\} = \text{given function of } r. \tag{2}$$

For ease of discussion, the bed is divided into N horizontal layers, N being chosen so large that vertical temperature differences within any layer are negligible.

Neglecting the heat capacity of the gas holdup, the heat balance of the gas for one layer can be written as

$$F_g\gamma_g T_{g,n-1} + Q_{s,n} = F_g\gamma_g T_{g,n} \tag{3}$$

where γ_g represents the specific heat of the gas (assumed constant). Further

$$\left.\frac{\partial T_{s,n}}{\partial r}\right|_{r=R} = -\left[T_{s,n}\{R, t\} - T_{a,n}\{t\}\right]\frac{U}{\lambda_s} \tag{4}$$